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Rolling tachyon boundary state, conserved charges and two dimensional string theory

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Abstract: The boundary state associated with the rolling tachyon solution on an unstable D-brane contains a part that decays exponentially in the asymptotic past and the asymptotic future, but it also contains other parts which either remain constant or grow exponentially in the past or future. We argue that the time dependence of the latter parts is completely determined by the requirement of BRST invariance of the boundary state, and hence they contain information about certain conserved charges in the system. We also examine this in the context of the unstable D0-brane in two dimensional string theory where these conseved charges produce closed string background associated with the discrete states, and show that these charges are in one to one correspondence with the symmetry generators in the matrix model description of this theory.

Keywords: Tachyon Condensation, D-branes.

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Contents

1. Introduction and summary

Unstable D-branes in bosonic and superstring theory admit time dependent solutions describing the rolling of the tachyon away from the maximum of the potential [1, 2, 3]. At the open string tree level these solutions are described by solvable boundary conformal field theories. Using this description one can calculate the sources for various closed string states produced by this rolling tachyon solution. The information about these sources can be summarized in the boundary state associated to the solution, which is a ghost number 3 state in the Hilbert space of closed string states.

The boundary state associated with the rolling tachyon solution can be divided into two parts. The first part gives rise to sources for various closed string states which fall off exponentially both in the asymptotic future and in the asymptotic past. Thus this part induces a closed string background that satisfy source free closed string field equations in the asymptotic past and the asymptotic future. Computation of total energy stored in the closed string field shows that while the amount of energy stored in a given mode of a fixed mass is small compared to the total energy of the D-brane in the weak coupling limit, the total amount of energy stored in the closed string field becomes infinite when we sum over all the modes [4, 5, 6]. Naively, this suggests that the tree level open string description of the system breaks down due to the backreaction of the closed string emission. However, a different viewpoint, proposed in [7, 8, 9], is that the closed strings do not invalidate the open string results, but simply provide a dual description of the same results. From this

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viewpoint, quantum open string theory provides a complete description of the unstable D-brane system, and there is no need to include the effect of closed string emission in the open string analysis. Ehrenfest theorem will then tell us that in the weak coupling limit tree level open string theory provides a complete understanding of the dynamics of the system. Evidence for this conjecture comes from the observation that many of the properties of the final system predicted by the tree level open string analysis, e.g. vanishing pressure and dilaton charge, agree with the properties of the final closed string field configuration in the weak coupling limit.

This conjecture can be put on a firm footing in two dimensional string theory based on the conformal field theory of a time like scalar field with central charge 1, and the Liouville field theory of central charge 25 [10, 11, 12].[1] This theory admits an unstable D0-brane and rolling tachyon solutions on this D0-brane. On the other hand this string theory has a dual description as a matrix model [15, 16, 17], which, in turn is described by a theory of free fermions in an inverted harmonic oscillator potential. The vacuum of this theory is described by a state where all states below a given level (fermi level) are filled, and all states above this level are empty. By expressing the closed string state produced by the rolling tachyon configuration in the language of this free fermion field theory one finds that this represents a state where a single fermion is excited from the fermi level to some energy above the fermi level [18, 19, 20, 13, 14, 21, 22, 23, 24, 25, 26, 27, 28, 29]. Thus the dynamics of a D0-brane is described completely by the theory of a single particle moving in an inverted harmonic oscillator potential, with the additional constraint that the energy levels of the system below the fermi level are removed by hand. This can be regarded as the exact open string description of the system. The closed string fields in this theory are obtained by bosonizing the free field theory of fermions in the inverted harmonic oscillator potential [30, 31, 32]. This clearly demonstrates that the closed strings provide a description that is dual to the open string description, but the open string theory is capable of providing a complete description of the system.

All this analysis has been done with only one part of the boundary state associated with the rolling tachyon solution. But both in the critical string theory and in the two dimensional string theory the boundary state has another part which gives rise to sources which do not vanish in the asymptotic past of future, but either remain constant [3] or grow exponentially [33] either in the past or in the future. Naively, this would again indicate that these exponentially growing closed string fields invalidate the classical open string description of the system. However the open-closed string duality conjecture stated above would lead one to believe that this is not so. What this may be indicating is an inadequacy in the closed string description rather than in the open string description. As an analogy we can cite the example of closed string field configurations produced by static stable D-branes. Often the field configuration hits a singularity near the core of the brane. However we do not take this as an indication of the breakdown of the open string description. Instead it is a reflection of the inadequacy of the closed string description.

[1] This has also been generalized to two dimensional string theories with world-sheet supersymmetry [13, 14], but for simplicity we shall focus our attention on two dimensional bosonic string theory only.

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It still makes sense however to explore whether the exponentially growing source terms contain any physical information about the system. In this paper we argue that they contain information about some conserved charges. The argument relies on the fact that while the requirement of BRST invariance does not put any constraint on the first part of the boundary state, it fixes the time dependence of the various source terms coming from the second part. While in the critical string theory we do not have any independent way of verifying these conservation laws, in two dimensional string theory we can find additional support for this interpretation by identifying these conserved charges in the matrix model description.

The rest of the paper is organised as follows. In section 2 we split the boundary state of the critical string theory into two parts each of which is separately BRST invariant. The first part vanishes in the asymptotic past and asymptotic future, but the second part contains constant as well as exponentially growing terms. We show that whereas the time dependence of the first part is not fixed by the requirement of BRST invariance of the boundary state, the time dependence of the second part does get fixed by this requirement. This leads to the suggestion that this part contains information about conserved charges of this system [3, 34]. These conserved charges are shown to be labeled by SU(2) quantum numbers (j, m) with the restriction $-(j-1) \leq m \leq (j-1)$. We also find the dependence of these charges on the parameter characterizing the rolling tachyon solution.

In section 3 we analyze the closed string field configuration produced by different parts of the boundary state. Since the first part vanishes in the limit $x^0 \to \pm\infty$, this produces source free on-shell closed string background in these limits [4, 5, 6]. For the second part the source terms do not vanish in the $x^0 \to \pm\infty$ limit. However, since the sources are localized at the location of the original D-brane, we get source free on-shell closed string background away from the location of the brane.

In section 4 we repeat the analysis of section 2 for the D0-brane of the two dimensional string theory. We split the boundary state associated with the rolling tachyon solution on the D0-brane into two parts each of which is separately BRST invariant, and following arguments identical to that in the case of critical string theory we show that while the first part vanishes in the asymptotic past and future, the second term contains information about conserved charges in the system. In section 5 we analyze the closed string fields produced by this boundary state. The first part produces on-shell closed string tachyon field in the asymptotic past and future [19], whereas the second part produces on-shell closed string background which are analytic continuation of the discrete states [35, 36, 37] in the euclidean two dimensional string theory.

Finally in section 6 we identify these conserved charges in the matrix model description of the two dimensional string theory. We begin this section with a review of the description of the D0-brane in the matrix model. We then explicitly identify a set of conserved charges in the matrix model following [31, 38, 39, 36, 40, 37, 41, 42, 43], and find the precise relation between these conserved chrges and those in the continuum description by comparing their dependences on the time coordinate x^0 and the parameter λ labelling the rolling tachyon solution. In particular we establish a one to one correspondence between the conserved charges in the two descriptions following this line of argument.

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Each of the sections also contains a large amount of material where we review the relevant aspects of the decaying part of the boundary state before turning our attention to the constant and the exponentially growing parts.

2. Boundary state for the rolling tachyon in critical string theory

We begin with a D-p-brane in critical bosonic string theory in flat (25+1) dimensional space-time. The rolling tachyon solution, parametrized by the constant λ, is obtained by deforming the conformal field theory describing the D-p-brane by a boundary term [2, 3]

$$\lambda \int dt \, \cosh(X^0(t)) \,. \tag{2.1}$$

We are using $\alpha' = 1$ unit. Under a Wick rotation $X^0 \to iX$, this becomes:

$$\lambda \int dt \, \cos(X(t)) \,. \tag{2.2}$$

The boundary state associated with the euclidean D-brane, corresponding to the deformation (2.2), is given by:

$$|\mathcal{B}\rangle = \mathcal{T}_p \, |\mathcal{B}\rangle_{c=1} \otimes |\mathcal{B}\rangle_{c=25} \otimes |\mathcal{B}\rangle_{\rm ghost} \,, \tag{2.3}$$

where $\mathcal{T}_p$ is the tension of the D-p-brane, $|\mathcal{B}\rangle_{c=1}$ denotes the boundary state associated with the X direction, $|\mathcal{B}\rangle_{c=25}$ denotes the boundary state associated with the other 25 directions $X^1, \ldots, X^{25}$, and $|\mathcal{B}\rangle_{\rm ghost}$ denotes the boundary state associated with the ghost direction. We have:

$$|\mathcal{B}\rangle_{c=25} = \int \frac{d^{25-p}k_\perp}{(2\pi)^{25-p}} \exp\left(\sum_{s=1}^{25} \sum_{n=1}^{\infty} \frac{1}{n} (-1)^{d_s} \, \alpha^s_{-n} \bar{\alpha}^s_{-n} \right) |k_{||} = 0, k_\perp\rangle \,. \tag{2.4}$$

and

$$|\mathcal{B}\rangle_{\rm ghost} = \exp\left(- \sum_{n=1}^{\infty} (\bar{b}_{-n} c_{-n} + b_{-n} \bar{c}_{-n}) \right) (c_0 + \bar{c}_0) c_1 \bar{c}_1 |0\rangle \,. \tag{2.5}$$

Here $d_s = 1$ if X^s has Neumann boundary condition and 0 if X^s has Dirichlet boundary condition, $\vec{k}_{||}$ denotes spatial momentum along the D-p-brane, $k_\perp$ denotes momentum transverse to the D-p-brane, α^s_n, $\bar{\alpha}^s_n$ denote the oscillators associated with the world-sheet scalar field X^s and b_n, $\bar{b}_n$, c_n, $\bar{c}_n$ denote the ghost oscillators. For $\lambda = 0$, $|\mathcal{B}\rangle_{c=1}$ becomes the standard boundary state for X with Neumann boundary condition:

$$|\mathcal{B}\rangle_{c=1}|_{\lambda=0} = \exp\left(- \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \bar{\alpha}_{-n} \right) |k = 0\rangle \,, \tag{2.6}$$

where k labels momentum along X.

For non-zero λ, it is convenient to express $|\mathcal{B}\rangle_{c=1}$ as a sum of two terms [44, 45, 3, 33, 46, 47, 48]:

$$|\mathcal{B}\rangle_{c=1} = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \bar{\alpha}_{-n} \right) f(X(0)) \, |0\rangle + |\widetilde{\mathcal{B}}\rangle_{c=1} \,, \tag{2.7}$$

where

$$f(x) = \frac{1}{1+\sin(\pi\lambda)e^{ix}} + \frac{1}{1+\sin(\pi\lambda)e^{-ix}} - 1\,,$$
$$= \sum_{n\in Z} (-1)^n \sin^{|n|}(\pi\lambda)\, e^{inx}\,. \tag{2.8}$$

Note the difference in sign in the exponents of (2.6) and (2.7). For any momentum k, $\exp\left(\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}\bar{\alpha}_{-n}\right)|k\rangle$ is annihilated by $L_n^X - \bar{L}_{-n}^X$ where L_n^X and $\bar{L}_n^X$ denote the Virasoro generators of the $c=1$ conformal field theory. Since $|\mathcal{B}\rangle_{c=1}$ must be annihilated by $L_n^X - \bar{L}_{-n}^X$, it follows that $|\widetilde{\mathcal{B}}\rangle_{c=1}$ must also be annihilated by $L_n^X - \bar{L}_{-n}^X$ and hence must be a linear combination of the Ishibashi states [49] built upon various left-right symmetric primary states in the $c=1$ conformal field theory. These primaries are labelled by the SU(2) quantum numbers (j,m) $(-j \le m \le j,\ j-m$ integer), with $2m$ denoting the X momentum carried by the state, and (j^2, j^2) being the conformal weight of the state. Thus the primary state $|j,m\rangle$ has the form:

$$|j,m\rangle = \widehat{\mathcal{P}}_{j,m}\, e^{2\,i\,m\,X(0)}|0\rangle\,, \tag{2.9}$$

where $\widehat{\mathcal{P}}_{j,m}$ is some combination of the X oscillators of level (j^2-m^2, j^2-m^2). We shall normalize $\widehat{\mathcal{P}}_{j,m}$ such that when we express $\exp\left(\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}\bar{\alpha}_{-n}\right)e^{2imX(0)}|0\rangle$ as a linear combination of the Ishibashi states built on various primaries, the Ishibashi state $|j,m\rangle\rangle$ built on the primary $|j,m\rangle$ appears with coefficient 1:

$$\exp\left(\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}\bar{\alpha}_{-n}\right)e^{2imX(0)}|0\rangle = \sum_{j\ge|m|}|j,m\rangle\rangle\,. \tag{2.10}$$

The complete contribution to $|\mathcal{B}\rangle_{c=1}$ from the Ishibashi states built over the primaries $|j,\pm j\rangle$, as well as part of the contribution from the other Ishibashi states, are included in the first term on the right hand side of (2.7). Thus the second term must be a linear combination of Ishibashi states built on $|j,m\rangle$ with $m \ne \pm j$:

$$|\widetilde{\mathcal{B}}\rangle_{c=1} = \sum_j \sum_{m=-j+1}^{j-1} f_{j,m}(\lambda)\,|j,m\rangle\rangle\,, \tag{2.11}$$

where $f_{j,m}(\lambda)$ are some functions of the parameter λ. These are given by [44, 45]:

$$f_{j,m}(\lambda) = D^j_{m,-m}(2\pi\lambda)\,\frac{(-1)^{2m}}{D^j_{m,-m}(\pi)} - (-1)^{2m}\,\sin^{2|m|}(\pi\lambda)\,, \tag{2.12}$$

where $D^j_{m,m'}(\theta)$ are the representation matrices of the SU(2) group element $e^{i\theta\sigma_1/2}$ in the spin j representation. The second term in (2.12) represents the effect of subtracting from $|\mathcal{B}\rangle_{c=1}$ the contribution due to the first term in (2.7). The fact that the total contribution to $|\mathcal{B}\rangle_{c=1}$ from $|j,m\rangle\rangle$ is proportional to $D^j_{m,-m}(2\pi\lambda)$ was shown in [44, 45]. The constant of proportionality is found by using the condition

$$f_{j,m}\left(\frac{1}{2}\right) = 0\,. \tag{2.13}$$

This relation arises as follows. As $\lambda \to \frac{1}{2}$, the system approaches an array of D-branes with Dirichlet boundary condition on X, situated at $x = (2k+1)\pi$ for integer k. On the other hand, from (2.8) one can show that

$$\lim_{\lambda \to \frac{1}{2}} f(x) = 2\pi \sum_{k \in Z} \delta\left(x - (2k+1)\,\pi\right). \tag{2.14}$$

In this case the first term in (2.7) reproduces the complete contribution to the boundary state, and hence the second term must vanish. This is the reason why the functions $f_{j,m}(\lambda)$ must vanish in the $\lambda \to \frac{1}{2}$ limit. For later use we quote here the form of $f_{j,m}(\lambda)$ for some specific (j, m):

$$f_{1,0}(\lambda) = -2\cos^2(\pi\lambda)\,, \qquad f_{\frac{3}{2},\frac{1}{2}}(\lambda) = 3\sin(\pi\lambda)\cos^2(\pi\lambda)\,, \tag{2.15}$$

etc.

The boundary state in the Minkowski theory with boundary interaction (2.1) is then obtained by the replacement $X \to -iX^0$ in the euclidean boundary state. If $|\widehat{\mathcal{B}}\rangle_{c=1}$ denotes the continuation of $|\widetilde{\mathcal{B}}\rangle_{c=1}$ to Minkowski space,

$$|\widehat{\mathcal{B}}\rangle_{c=1} = |\widetilde{\mathcal{B}}\rangle_{c=1}|_{X \to -iX^0}\,, \tag{2.16}$$

then the complete boundary state in the Minkowski space is given by:

$$|\mathcal{B}\rangle = |\mathcal{B}_1\rangle + |\mathcal{B}_2\rangle\,, \tag{2.17}$$

where

$$\begin{aligned} |\mathcal{B}_1\rangle \;=&\; \mathcal{T}_p \exp\left(-\sum_{n=1}^{\infty} \frac{1}{n}\alpha^0_{-n}\bar{\alpha}^0_{-n}\right) \widetilde{f}(X^0(0))\,|0\rangle \otimes \\ &\otimes \int \frac{d^{25-p}k_\perp}{(2\pi)^{25-p}} \exp\left(\sum_{n=1}^{\infty}\sum_{s=1}^{25}(-1)^{d_s}\,\frac{1}{n}\,\alpha^s_{-n}\bar{\alpha}^s_{-n}\right)|\vec{k}_{||} = 0, \vec{k}_\perp\rangle \otimes \\ &\otimes \exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n} + b_{-n}\bar{c}_{-n})\right)(c_0+\bar{c}_0)c_1\bar{c}_1|0\rangle\,, \end{aligned} \tag{2.18}$$

and

$$\begin{aligned} |\mathcal{B}_2\rangle \;=&\; \mathcal{T}_p\,|\widehat{\mathcal{B}}\rangle_{c=1} \otimes \int \frac{d^{25-p}k_\perp}{(2\pi)^{25-p}} \exp\left(\sum_{n=1}^{\infty}\sum_{s=1}^{25}(-1)^{d_s}\,\frac{1}{n}\,\alpha^s_{-n}\bar{\alpha}^s_{-n}\right)|\vec{k}_{||} = 0, \vec{k}_\perp\rangle \otimes \\ &\otimes \exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n} + b_{-n}\bar{c}_{-n})\right)(c_0+\bar{c}_0)c_1\bar{c}_1|0\rangle\,, \end{aligned} \tag{2.19}$$

where

$$\widetilde{f}(x^0) = f(-ix^0) = \frac{1}{1+\sin(\pi\lambda)e^{x^0}} + \frac{1}{1+\sin(\pi\lambda)e^{-x^0}} - 1\,. \tag{2.20}$$

Using the fact that $L^{X^0}_n - \bar{L}^{X^0}_{-n}$ annihilates $\exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\alpha^0_{-n}\bar{\alpha}^0_{-n}\right)|k^0\rangle$ it is easy to verify that $|\mathcal{B}_1\rangle$ is BRST invariant, i.e.

$$(Q_B + \bar{Q}_B)|\mathcal{B}_1\rangle = 0\,. \tag{2.21}$$

Indeed we have the stronger relation

$$
\begin{aligned}
(Q_B + \bar{Q}_B) \Bigg[\exp\left(- \sum_{n=1}^{\infty} \frac{1}{n} \alpha^0_{-n} \bar{\alpha}^0_{-n} \right) |k^0\rangle \otimes \\
\otimes \exp\left(\sum_{n=1}^{\infty} \sum_{s=1}^{25} (-1)^{d_s} \frac{1}{n} \alpha^s_{-n} \bar{\alpha}^s_{-n} \right) |\vec{k}_{||} = 0, \vec{k}_\perp\rangle \otimes \\
\otimes \exp\left(- \sum_{n=1}^{\infty} (\bar{b}_{-n} c_{-n} + b_{-n} \bar{c}_{-n}) \right) (c_0 + \bar{c}_0) c_1 \bar{c}_1 |0\rangle \Bigg] = 0 \,,
\end{aligned}
\tag{2.22}
$$

for any k^0 and $\vec{k}_\perp$. Since $|\mathcal{B}\rangle = |\mathcal{B}_1\rangle + |\mathcal{B}_2\rangle$ is BRST invariant, and $|\mathcal{B}_1\rangle$ is BRST invariant, we see that $|\mathcal{B}_2\rangle$ is also BRST invariant:

$$
(Q_B + \bar{Q}_B) |\mathcal{B}_2\rangle = 0 \,. \tag{2.23}
$$

This also follows directly from the fact that $L_n^{X^0} - \bar{L}_{-n}^{X^0}$ annihilates $|\widehat{\mathcal{B}}\rangle_{c=1}$.

Let us define $\widehat{A}_N$ to be an operator of level (N, N), composed of negative moded oscillators of X^0, X^s, b, c, $\bar{b}$ and $\bar{c}$ such that

$$
\exp\left[\sum_{n=1}^{\infty} \left(-\frac{1}{n} \alpha^0_{-n} \bar{\alpha}^0_{-n} + \sum_{s=1}^{25} (-1)^{d_s} \frac{1}{n} \alpha^s_{-n} \bar{\alpha}^s_{-n} - (\bar{b}_{-n} c_{-n} + b_{-n} \bar{c}_{-n}) \right) \right] = \sum_{N=0}^{\infty} \widehat{A}_N \,. \tag{2.24}
$$

Here $\widehat{A}_0 = 1$. Then $|\mathcal{B}_1\rangle$ can be expressed as

$$
|\mathcal{B}_1\rangle = \mathcal{T}_p \int \frac{d^{25-p} k_\perp}{(2\pi)^{25-p}} \sum_{N=0}^{\infty} \widehat{A}_N \, (c_0 + \bar{c}_0) \, c_1 \, \bar{c}_1 \, \widetilde{f}\left(X^0(0) \right) \, |k^0 = 0, \vec{k}_{||} = 0, \vec{k}_\perp\rangle \,. \tag{2.25}
$$

Also since Q_B and $\bar{Q}_B$ preserves the level of a state, (2.22) and (2.24) give

$$
(Q_B + \bar{Q}_B) \, \widehat{A}_N \, (c_0 + \bar{c}_0) \, c_1 \, \bar{c}_1 \, |k^0, \vec{k}_{||} = 0, \vec{k}_\perp\rangle = 0 \,. \tag{2.26}
$$

We shall make use of these relations later.

From (2.9), (2.11) we see that $|\widetilde{\mathcal{B}}\rangle_{c=1}$ is built on states carrying integer x momentum. Upon continuation to the Minkowski space these correspond to states built on $e^{nX(0)}|0\rangle$ for integer n. (2.19) then allows us to express $|\mathcal{B}_2\rangle$ as

$$
|\mathcal{B}_2\rangle = \mathcal{T}_p \sum_{n=-\infty}^{\infty} \sum_{N=1}^{\infty} \int \frac{d^{25-p} k_\perp}{(2\pi)^{25-p}} \, \widehat{\mathcal{O}}_N^{(n)} \, (c_0 + \bar{c}_0) \, c_1 \, \bar{c}_1 \, e^{nX^0(0)} \, |k^0 = 0, \vec{k}_{||} = 0, \vec{k}_\perp\rangle \,, \tag{2.27}
$$

where $\widehat{\mathcal{O}}_N^{(n)}$ is some fixed combination of negative moded oscillators of total level (N, N). The structure of $\widehat{\mathcal{O}}_N^{(n)}$ is different for different n since the primaries $|j, m\rangle$ for $m \neq \pm j$ involve different oscillator combinations for different (j, m).[2] Note that the sum over N starts at 1, since in the conformal field theory involving the X^0 field $|\mathcal{B}_2\rangle$ involves Virasoro

[2]This is apparent from the fact that the level of the oscillator combination in $\widehat{P}_{j,m}$ in (2.9) is $(j^2 - m^2)$ which clearly depends on j and m for $m \neq \pm j$.

descendants of primaries $|j,m\rangle$ of level $(j^2-m^2, j^2-m^2) \geq (1,1)$. Since $(Q_B+\bar{Q}_B)$ preserves the momenta as well as the level of a state, we can conclude from (2.23), (2.27) that

$$(Q_B+\bar{Q}_B)\,\widehat{\mathcal{O}}_N^{(n)}\,(c_0+\bar{c}_0)\,c_1\,\bar{c}_1\,e^{nX^0(0)}\,|k^0=0,\vec{k}_\parallel=0,\vec{k}_\perp\rangle=0\,. \tag{2.28}$$

We now note some crucial differences between the structure of $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_2\rangle$. Since the function $\widetilde{f}(x^0)$ defined in (2.20) vanishes as $x^0\to\pm\infty$, the source terms for the closed string fields produced by $|\mathcal{B}_1\rangle$ vanish asymptotically. In contrast the source associated with the n-th term in the sum in (2.27) is proportional to e^{nx^0}, and grows for $x^0\to\infty$ ($x^0\to-\infty$) for positive (negative) n. The other crucial difference between $|\mathcal{B}_1\rangle$ and $\mathcal{B}_2\rangle$ is that while the requirement of BRST invariance does not give any constraint on the time dependence of the source terms generated by $|\mathcal{B}_1\rangle$, it completely fixes the time dependence of the source terms generated by $|\mathcal{B}_2\rangle$. To see this we note that if we replace $\widetilde{f}(x^0)$ by any arbitrary function in the expression (2.18) of $|\mathcal{B}_1\rangle$, we shall still get a BRST invariant state due to eq. (2.22). On the other hand, if we replace the factor $e^{nX^0(0)}$ in the expression (2.27) of $|\mathcal{B}_2\rangle$ by an arbitrary function $g^{(n)}(X^0(0))$, $|\mathcal{B}_2\rangle$ ceases to be BRST invariant. To see this we use eqs. (2.11), (2.16), (2.19) to express $|\mathcal{B}_2\rangle$ in a form slightly different from that given in (2.27):

$$|\mathcal{B}_2\rangle=\mathcal{T}_p\sum_{j=1}^{\infty}\sum_{m=-(j-1)}^{j-1}\int\frac{d^{25-p}k_\perp}{(2\pi)^{25-p}}\,f_{j,m}(\lambda)\,\widehat{\mathcal{R}}_{j,m}\,(c_0+\bar{c}_0)c_1\bar{c}_1e^{2mX^0(0)}\,|k^0=0,\vec{k}_\parallel=0,\vec{k}_\perp\rangle\,, \tag{2.29}$$

where

$$\widehat{\mathcal{R}}_{j,m}=\widehat{\mathcal{N}}_{j,m}\,\exp\left(\sum_{n=1}^{\infty}\sum_{s=1}^{25}(-1)^{d_s}\,\frac{1}{n}\,\alpha^s_{-n}\bar{\alpha}^s_{-n}\right)\,\exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n}+b_{-n}\bar{c}_{-n})\right). \tag{2.30}$$

$\widehat{\mathcal{N}}_{j,m}$ in turn is an operator made of α^0_{-n}, $\bar{\alpha}^0_{-n}$ for $n>0$ such that the Ishibashi state $|j,m\rangle\rangle$ in the Minkowski theory is given by:

$$|j,m\rangle\rangle=\widehat{\mathcal{N}}_{j,m}\,e^{2mX^0(0)}\,|0\rangle\,. \tag{2.31}$$

From eqs. (2.30), (2.31) it is clear that $\widehat{\mathcal{R}}_{j,m}$ does not have any explicit λ dependence. Now consider generalizing the source terms given by $|\mathcal{B}_2\rangle$ in a way that preserves the operator structure but gives the source terms arbitrary time dependence:

$$|\mathcal{B}_2\rangle'=\mathcal{T}_p\sum_{j=1}^{\infty}\sum_{m=-(j-1)}^{j-1}\int\frac{d^{25-p}k_\perp}{(2\pi)^{25-p}}\,\widehat{\mathcal{R}}_{j,m}\,(c_0+\bar{c}_0)c_1\bar{c}_1g_{j,m}(X^0(0))\,|k^0=0,\vec{k}_\parallel=0,\vec{k}_\perp\rangle\,. \tag{2.32}$$

Requiring the BRST invariance of $|\mathcal{B}_2\rangle'$

$$(Q_B+\bar{Q}_B)|\mathcal{B}_2\rangle'=0\,, \tag{2.33}$$

we get

$$\partial_0\left(e^{-2mx^0}g_{j,m}(x^0)\right)=0\,. \tag{2.34}$$

This follows from the fact that for generic $g_{j,m}(x^0)$, the state $\widehat{\mathcal{P}}_{j,m}\, g_{j,m}(X^0(0))\,|0\rangle$ is no longer a primary state, and more generally $\widehat{\mathcal{N}}_{j,m}\, g_{j,m}(X^0(0))\,|0\rangle$ is no longer an Ishibashi state. Thus there will be additional contributions from the $(c_{-n}L_n^{X^0} + \bar{c}_{-n}\bar{L}_n^{X^0})$ terms in $Q_B + \bar{Q}_B$ acting on this state. These additional contributions will vanish when $g_{j,m}(x^0)$ satisfy (2.34).

The general arguments given in refs. [3, 34] as well as the explicit form of eq. (2.34) suggests that $e^{-2mx^0} g_{j,m}(x^0)$ can be thought of as a conserved charge.[3] From this we see that the conserved charges are characterized by two half integers (j, m) in the range $j \geq 1$, $-(j-1) \leq m \leq (j-1)$. Comparing (2.29) and (2.32) we see that for the boundary state $|\mathcal{B}_2\rangle$ associated with the rolling tachyon solution, we have

$$e^{-2mx^0} g_{j,m}(x^0) = f_{j,m}(\lambda)\,. \tag{2.35}$$

(2.13) shows that all these charges vanish at $\lambda = \frac{1}{2}$. This is expected since $\lambda = \frac{1}{2}$ describes the closed string vacuum without any D-brane. For $j = 1$, $m = 0$ the conserved charge is proportional to the energy density of the D-brane [3].

We end this section with a cautionary remark. The analysis given here shows that the charges $g_{j,m}(x^0)$ are conserved at least in a subsector of the open string theory which corresponds to adding boundary perturbation involving X^0 and its derivatives, since in this case the final boundary state will have the product structure $|\mathcal{B}\rangle_{c=1}\otimes|\mathcal{B}\rangle_{c=25}\otimes|\mathcal{B}\rangle_{\rm ghost}$, with $|\mathcal{B}\rangle_{c=1}$ given by some linear combination of the Ishibashi states in the $c = 1$ conformal field theory. Thus $g_{j,m}(x^0)$ can be defined and shown to be conserved following the procedure outlined in this section. Whether these conservation laws have analogs in the full open string theory remains to be seen. Nevertheless having conservation laws of this type even in a restricted subsector of the theory could facililate analysis of classical solutions in that subsector. In section 6 we shall see that in the two dimensional string theory these conservation laws do hold in the full theory.

3. Closed string field produced by the rolling tachyon in critical string theory

The closed string field $|\Psi_c\rangle$ is a state of ghost number 2 in the Hilbert space of first quantized closed string theory, satisfying the constraint [50]

$$b_0^-|\Psi_c\rangle = 0\,, \qquad L_0^-|\Psi_c\rangle = 0\,, \tag{3.1}$$

where we define

$$c_0^{\pm} = (c_0 \pm \bar{c}_0)\,, \qquad b_0^{\pm} = (b_0 \pm \bar{b}_0)\,, \qquad L_0^{\pm} = (L_0 \pm \bar{L}_0)\,. \tag{3.2}$$

c_n, $\bar{c}_n$, b_n, $\bar{b}_n$ are the usual ghost oscillators, and L_n, $\bar{L}_n$ are the total Virasoro generators. The quadratic part of the closed string field theory action is given by:

$$-\frac{1}{Kg_s^2}\langle\Psi_c|c_0^-(Q_B + \bar{Q}_B)|\Psi_c\rangle\,, \tag{3.3}$$

[3]Note that for each pair (j, m) with $j \geq 1$, $-(j-1) \leq m \leq (j-1)$, we can in principle define a conserved charge for every primary of the $c = 25$ CFT, since the action of $(Q_B + \bar{Q}_B)$ does not mix the Verma modules built over such primaries. However all these charges will be proportional to $f_{j,m}(\lambda)$.

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where Q_B and $\bar{Q}_B$ are the holomorphic and anti-holomorphic components of the BRST charge, K is a normalization constant to be given in eq. (3.6), and g_s is the closed string coupling constant. In the presence of the D-brane we need to add an extra source term to the action:

$$\langle \Psi_c | c_0^- | \mathcal{B} \rangle \,. \tag{3.4}$$

The equation of motion of $|\Psi_c\rangle$ is then

$$2\,(Q_B + \bar{Q}_B)\,|\Psi_c\rangle = K\, g_s^2\, |\mathcal{B}\rangle \,. \tag{3.5}$$

Clearly by a rescaling of $|\Psi_c\rangle$ we can change K and the normalization of $|\mathcal{B}\rangle$. However once the normalization of $|\mathcal{B}\rangle$ is fixed in a convenient manner (as in eq. (2.3)), the normalization constant K can be determined by requiring that in the euclidean theory the classical action, obtained after eliminating $|\Psi_c\rangle$ using its equation of motion (3.5) and substituting it back in the sum of (3.3) and (3.4), reproduces the one loop partition function $Z_{\rm open}$ of the open string theory on the D-brane. Using the solution to (3.5) given in (3.7) we get

$$Z_{\rm open} = \frac{1}{2}\, K\, g_s^2\, \langle \mathcal{B} | (c_0 - \bar{c}_0)\, [2(L_0 + \bar{L}_0)]^{-1}\, (b_0 + \bar{b}_0)\, |\mathcal{B}\rangle \,. \tag{3.6}$$

Since $Z_{\rm open}$ is independent of g_s, and $|\mathcal{B}\rangle$ is inversely proportional to g_s due to the $\mathcal{T}_p$ factor in (2.3), we see that K is a purely numerical constant.

We want to look for solutions to eq. (3.5). Noting that $|\mathcal{B}\rangle$ is BRST invariant, and that $\{Q_B + \bar{Q}_B, b_0 + \bar{b}_0\} = (L_0 + \bar{L}_0)$, we can write down a solution to equation (3.5) as:

$$|\Psi_c\rangle = K\, g_s^2\, [2(L_0 + \bar{L}_0)]^{-1}\, (b_0 + \bar{b}_0)\, |\mathcal{B}\rangle \,. \tag{3.7}$$

This solution satisfies the Siegel gauge condition $(b_0 + \bar{b}_0)|\Psi_c\rangle = 0$. We can of course construct other solutions which are gauge equivalent to this one by adding to $|\Psi_c\rangle$ terms of the form $(Q_B + \bar{Q}_B)|\Lambda\rangle$. However even within Siegel gauge, the right hand side of (3.7) is not defined unambiguously. Since free closed string field theory in Minkowski space has infinite number of plane wave solutions in the Siegel gauge, satisfying

$$(L_0 + \bar{L}_0)|\Psi_c\rangle = 0\,, \qquad (b_0 + \bar{b}_0)|\Psi_c\rangle = 0\,, \tag{3.8}$$

the right hand side of (3.7) is defined only up to addition of solutions of (3.8). However, since a solution of (3.8) does not in general satisfy the full set of source free string field field equations of motion $(Q_B + \bar{Q}_B)|\Psi_c\rangle = 0$, addition of an arbitrary solution of (3.8) to a solution to (3.5) will not, in general, generate a solution of (3.5). Thus we need to carefully choose a prescription for defining the right hand side of (3.7) in order to construct a solution of eq. (3.5). A natural prescription (known as the Hartle-Hawking prescription) is to begin with the solution of the associated equations of motion in the euclidean theory where there is a unique solution to eq. (3.7) (which therefore satisfies the full equation (3.5)) and then analytically continue the result to the Minkowski space along the branch passing through the origin $x^0 = 0$ [4, 5, 6]. This is the prescription we shall follow.

Since the boundary state $|\mathcal{B}\rangle$ for the rolling tachyon configuration can be regarded as a sum of two components $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_2\rangle$ each of which are separately gauge invariant, we shall analyze their effects separately. Let us denote by $|\Psi_c^{(1)}\rangle$ and $|\Psi_c^{(2)}\rangle$ the closed string field configurations produced by $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_2\rangle$ respectively. We begin with the analysis of $|\Psi_c^{(1)}\rangle$. The result for this is already contained implicitly in [4, 5, 6], but we shall reproduce these results for completeness. Let $\phi_n(\vec{k}, x^0)$ denote a closed string field with spatial momenta $\vec{k}$, appearing in the expansion of $|\Psi_c^{(1)}\rangle$ as the coefficient of a state for which the oscillator contribution to the $L_0 + \bar{L}_0$ eigenvalue is $m_n^2/2$. We shall call m_n the mass of ϕ_n even though ϕ_n may not represent a physical closed string state of mass m_n. Let $j_n(\vec{k}, x^0)$ be the source of $\phi_n(\vec{k}, x^0)$ appearing in the expansion of $(b_0 + \bar{b}_0)\,|\mathcal{B}_1\rangle$. Since $2(L_0 + \bar{L}_0)$ acting on $|\Psi_c\rangle$ has the effect of converting $\phi_n(\vec{k}, x^0)$ to $(\partial_0^2 + \vec{k}^2 + m_n^2)\phi_n(\vec{k}, x^0)$, the equations of motion satisfied by this field is given by:

$$(\partial_0^2 + \vec{k}^2 + m_n^2)\phi_n(\vec{k}, x^0) = K\, g_s^2\, j_n(\vec{k}, x^0)\,. \tag{3.9}$$

In the euclidean theory, obtained by replacing x^0 by ix, the equation takes the form:

$$(-\partial_x^2 + \vec{k}^2 + m_n^2)\, \phi_n(\vec{k}, ix) = K\, g_s^2\, j_n(\vec{k}, ix)\,. \tag{3.10}$$

This has solution:

$$\phi_n(\vec{k}, ix) = \frac{K\, g_s^2}{2\omega_n(\vec{k})} \left[\int_{-\infty}^{x} e^{-\omega_n(\vec{k})(x-x')} j_n(\vec{k}, ix')dx' + \int_{x}^{\infty} e^{\omega_n(\vec{k})(x-x')} j_n(\vec{k}, ix')dx' \right]. \tag{3.11}$$

where $\omega_n(\vec{k}) = \sqrt{\vec{k}^2 + m_n^2}$. In terms of the variable $x^0 = ix$, $x'^0 = ix'$, this may be written as

$$\begin{aligned}
\phi_n(\vec{k}, x^0) &= -\frac{i\, K\, g_s^2}{2\omega_n(\vec{k})} \left[\int_{-i\infty}^{x^0} e^{i\omega_n(\vec{k})(x^0 - x'^0)} j_n(\vec{k}, x'^0)dx'^0 + \right. \\
&\qquad \left. + \int_{x^0}^{i\infty} e^{-i\omega_n(\vec{k})(x^0 - x'^0)} j_n(\vec{k}, x'^0)dx'^0 \right] \\
&= \frac{i\, K\, g_s^2}{2\omega_n(\vec{k})} \left[\int_{i\infty}^{x^0} e^{-i\omega_n(\vec{k})(x^0 - x'^0)} j_n(\vec{k}, x'^0)dx'^0 - \right. \\
&\qquad \left. - \int_{-i\infty}^{x^0} e^{i\omega_n(\vec{k})(x^0 - x'^0)} j_n(\vec{k}, x'^0)dx'^0 \right],
\end{aligned} \tag{3.12}$$

for x^0 lying on the imaginary axis. We now define its analytic continuation to the real axis by analytically continuing $\phi_n(\vec{k}, x^0)$ near the origin along the real axis. This gives, for real x:

$$\phi_n(\vec{k}, x^0) = \frac{i\, K\, g_s^2}{2\omega_n(\vec{k})} \left[\int_C e^{-i\omega_n(\vec{k})(x^0 - x'^0)} j_n(\vec{k}, x'^0)dx'^0 - \int_{C'} e^{i\omega_n(\vec{k})(x^0 - x'^0)} j_n(\vec{k}, x'^0)dx'^0 \right], \tag{3.13}$$

where the contour C runs from $i\infty$ to the origin along the imaginary x'^0 axis, and then to x^0 along the real x'^0 axis, and the contour C' runs from $-i\infty$ to the origin along the imaginary x'^0 axis, and then to x^0 along the real x'^0 axis. These are known as the Hartle-Hawking contours.

The specific form of $j_n(\vec{k}, x^0)$ can be read out from the expansion (2.25) of $|\mathcal{B}_1\rangle$. The leven (N,N) term acts as a source for a closed string field of mass^2 $= 4(N-1) \equiv m_N^2$. Then using (2.25) and (3.13) we can express the closed string field $|\Psi_c^{(1)}\rangle$ produced by $|\mathcal{B}_1\rangle$ as:

$$|\Psi_c^{(1)}\rangle = 2\, K\, g_s^2\, \mathcal{T}_p \int \frac{d^{25-p}k_\perp}{(2\pi)^{25-p}} \sum_{N\geq 0} \widehat{A}_N\, h_{\vec{k}_\perp}^{(N)}(X^0(0))\, c_1\, \bar{c}_1\, |k_\| = 0, \vec{k}_\perp\rangle\,, \tag{3.14}$$

where

$$h_{\vec{k}_\perp}^{(N)}(x^0) = \frac{i}{2\omega_{\vec{k}_\perp}^{(N)}} \left[\int_C e^{-i\omega_{\vec{k}_\perp}^{(N)}(x^0 - x'^0)} \widetilde{f}(x'^0) dx'^0 - \int_{C'} e^{i\omega_{\vec{k}_\perp}^{(N)}(x^0 - x'^0)} \widetilde{f}(x'^0) dx'^0 \right], \tag{3.15}$$

with

$$\omega_{\vec{k}_\perp}^{(N)} = \sqrt{\vec{k}_\perp^2 + m_N^2} = \sqrt{\vec{k}_\perp^2 + 4(N-1)}\,. \tag{3.16}$$

The overall multiplicative factor of 2 in (3.14) is due to the factor of 2 produced by the anti-commutator of $(b_0 + \bar{b}_0)$ and $(c_0 + \bar{c}_0)$ in $(b_0 + \bar{b}_0)|\mathcal{B}_1\rangle$. In the $x^0 \to \infty$ limit we can evaluate the integrals by closing the contours C and C' in the first and the fourth quadrangles respectively. This gives:

$$h_{\vec{k}_\perp}^{(N)}(x^0 \to \infty) = \frac{\pi}{\sinh(\pi\omega_{\vec{k}_\perp}^{(N)})} \frac{1}{2\omega_{\vec{k}_\perp}^{(N)}} \left[e^{-i\omega_{\vec{k}_\perp}^{(N)}(x^0 + \ln\sin(\pi\lambda))} + e^{i\omega_{\vec{k}_\perp}^{(N)}(x^0 + \ln\sin(\pi\lambda))} \right]. \tag{3.17}$$

Substituting this into (3.14) we get the asymptotic form of $|\Psi_c^{(1)}\rangle$ in the $x^0 \to \infty$ limit to be:

$$\begin{aligned} |\Psi_c^{(1)}\rangle \to\ & 2\, K\, g_s^2\, \mathcal{T}_p \int \frac{d^{25-p}k_\perp}{(2\pi)^{25-p}} \sum_{N\geq 0} \frac{\pi}{\sinh(\pi\omega_{\vec{k}_\perp}^{(N)})} \frac{1}{2\omega_{\vec{k}_\perp}^{(N)}} \widehat{A}_N\, c_1\, \bar{c}_1 \times \\ & \times \Bigg[e^{-i\omega_{\vec{k}_\perp}^{(N)} \ln\sin(\pi\lambda)} |k^0 = \omega_{\vec{k}_\perp}^{(N)}, \vec{k}_\| = 0, \vec{k}_\perp\rangle + \\ & \quad + e^{i\omega_{\vec{k}_\perp}^{(N)} \ln\sin(\pi\lambda)} |k^0 = -\omega_{\vec{k}_\perp}^{(N)}, \vec{k}_\| = 0, \vec{k}_\perp\rangle \Bigg]. \end{aligned} \tag{3.18}$$

$|\Psi_c^{(1)}\rangle$ defined in (3.14) clearly satisfies the Siegel gauge equations of motion:

$$2(L_0 + \bar{L}_0)|\Psi_c^{(1)}\rangle = K\, g_s^2\, (b_0 + \bar{b}_0)|\mathcal{B}_1\rangle\,. \tag{3.19}$$

Let us now try to verify explicitly that (3.14) satisfies the full set of equations of motion (3.5) with $|\mathcal{B}\rangle$ replaced by $|\mathcal{B}_1\rangle$. For this we express $(Q_B + \bar{Q}_B)$ as a sum of two terms:

$$Q_B + \bar{Q}_B = (c_0 L_0 + \bar{c}_0 \bar{L}_0) + \widehat{Q}\,, \tag{3.20}$$

where $\widehat{Q}$ does not contain any c or $\bar{c}$ zero modes and hence anti-commute with b_0 and $\bar{b}_0$. (2.26) then gives:

$$\widehat{Q}\, \widehat{A}_N\, (c_0 + \bar{c}_0)\, c_1\, \bar{c}_1\, |k^0, \vec{k}_\| = 0, \vec{k}_\perp\rangle = 0\,, \tag{3.21}$$

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where we have used the fact that $(L_0 - \bar{L}_0)$ annihilates $\widehat{A}_N\,(c_0+\bar{c}_0)\,c_1\,\bar{c}_1\,|k^0, \vec{k}_{||}=0, \vec{k}_\perp\rangle$. Applying $(b_0+\bar{b}_0)$ on (3.21) and using the fact that $(b_0+\bar{b}_0)$ anti-commutes with $\widehat{Q}$ and commutes with $\widehat{A}_N$, we get:

$$\widehat{Q}\widehat{A}_N\,c_1\,\bar{c}_1\,|k^0, \vec{k}_{||}=0, \vec{k}_\perp\rangle = 0\,. \tag{3.22}$$

Using (3.19)–(3.22) we get:

$$2\,(Q_B+\bar{Q}_B)|\Psi_c^{(1)}\rangle = 2\,(c_0L_0+\bar{c}_0\bar{L}_0)|\Psi^{1)}\rangle = K\,g_s^2\,|\mathcal{B}_1\rangle\,, \tag{3.23}$$

as required.

The crucial relation leading to the final result is (3.22) which shows that $\widehat{Q}$ annihilates the closed string field configuration $|\Psi_c^{(1)}\rangle$. This in turn is a consequence of the fact that for a given momentum $(k^0, \vec{k})$ and a given level (N, N) the combination of oscillators $\widehat{A}_N$ that appears in the expression for $|\Psi^{(1)}\rangle$ is the same as the one that appears in the expression for $|\mathcal{B}_1\rangle$. This is a special property of the specific definition of $(L_0+\bar{L}_0)^{-1}$ through the Hartle-Hawking prescription that we have used and will not hold for a generic definition.[4] For example, we could have expressed the level N contribution to $|\mathcal{B}_1\rangle$ as a linear combination of some fixed basis of level (N, N) states, and used different prescription for $(L_0+\bar{L}_0)^{-1}$ for these different basis states (e.g. Hartle-Hawking prescription for some and retarded Greens function for the others). The result will be a level (N, N) state that involves an oscillator combination different from $\widehat{A}_N$, and would not be annihilated by $\widehat{Q}$.

Since $\widetilde{f}(x^0)$ vanishes as $x^0 \to \pm\infty$, in the far future and far past we are left with pure closed string background satisfying free field equations of motion $(Q_B+\bar{Q}_B)|\Psi_c^{(1)}\rangle = 0$, i.e. on-shell closed string field configuration. One amusing point to note is that (3.18) does not vanish even in the $\lambda \to \frac{1}{2}$ limit, even though the boundary state $|\mathcal{B}_1\rangle$ vanishes in this limit [5]. This is because in the euclidean theory the boundary state $|\mathcal{B}_1\rangle$ represents an array of D-branes with Dirichlet boundary condition on $X = -iX^0$, located at $x = (2n+1)\pi$. This produces a non-trivial background in the euclidean theory, which, upon inverse Wick rotation, produces a source free closed string background in the Minkowski theory [5, 6]. The other important point to note is that the dependence of $|\Psi_c^{(1)}\rangle$ on λ in the $x^0 \to \infty$ limit comes only through a λ dependent time delay of $-\ln(\sin(\pi\lambda))$ [4].

We now turn to the analysis of closed string fields generated by $|\mathcal{B}_2\rangle$. We begin with the form (2.27) of $|\mathcal{B}_2\rangle$. Since $\widehat{\mathcal{O}}_N^{(n)}\,(c_0+\bar{c}_0)\,c_1\,\bar{c}_1\,e^{nX^0(0)}\,|\vec{k}_\perp\rangle$ is an eigenstate of $2(L_0+\bar{L}_0)$ with eigenvalue $(4(N-1)+n^2+\vec{k}_\perp^2)$, the natural choice of the closed string field produced by $|\mathcal{B}_2\rangle$, as given in eq. (3.7), is

$$\begin{aligned}|\Psi_c^{(2)}\rangle = 2\,K\,g_s^2\,\mathcal{T}_p \sum_{n\in Z}\sum_{N=1}^{\infty}\int \frac{d^{25-p}k_\perp}{(2\pi)^{25-p}}\left(4(N-1)+n^2+\vec{k}_\perp^2\right)^{-1}\times\\ \times\widehat{\mathcal{O}}_N^{(n)}\,c_1\,\bar{c}_1\,e^{nX^0(0)}\,|k^0=0, \vec{k}_{||}=0, \vec{k}_\perp\rangle\,.\end{aligned} \tag{3.24}$$

Clearly, this is the result that we shall get if we begin with the closed string background produced by the boundary state in the euclidean theory and then analytically continue it

[4] Of course this does not mean that this is the only possible prescription.

to the Minkowski space. Following the same procedure as in the case of $|\Psi_c^{(1)}\rangle$ one can show that (3.24) satisfies the full string field equation:

$$2\,(Q_B+\bar{Q}_B)|\Psi_c^{(2)}\rangle = 2\,(c_0 L_0+\bar{c}_0\bar{L}_0)|\Psi^{(2)}\rangle = K\, g_s^2\, |\mathcal{B}_2\rangle\,. \tag{3.25}$$

The crucial relation that establishes the first equality in (3.25) is:

$$\widehat{Q}\,\widehat{\mathcal{O}}_N^{(n)}\, c_1\, \bar{c}_1\, |k^0, \vec{k}_{||}=0, \vec{k}_\perp\rangle\,, \tag{3.26}$$

which follows from (2.28).

The space-time interpretation of this state for a given value of n is that it represents a field which grows as e^{nx^0}. For positive n this diverges as $x^0\to\infty$ and for negative n this diverges as $x^0\to-\infty$. On the other hand in the transverse spatial directions the solution falls off as $G(\vec{x}_\perp, \sqrt{4(N-1)+n^2})$ where $G(\vec{x}_\perp, m)$ denotes the euclidean Greens function of a scalar field of mass m in $(25-p)$ dimensions. Since $G(\vec{x}_\perp, m)\sim e^{-m|\vec{x}_\perp|}/|\vec{x}_\perp|^{(24-p)/2}$ for non-zero m and large $|\vec{x}_\perp|$, we see that the coefficients of the closed string field associated with the state $\widehat{\mathcal{O}}_N^{(n)} c_1\bar{c}_1|k\rangle$ behaves as

$$\exp\left(nx^0-\sqrt{4(N-1)+n^2}\,|\vec{x}_\perp|\right)/|\vec{x}_\perp|^{(24-p)/2}\,. \tag{3.27}$$

Thus at any given time x^0, the field associated with $\widehat{\mathcal{O}}_N^{(n)} c_1\bar{c}_1|k\rangle$ is small for $|\vec{x}_\perp|\gg nx^0/\sqrt{4(N-1)+n^2}$ and large for $|\vec{x}_\perp|\ll nx^0/\sqrt{4(N-1)+n^2}$. We can view such a field configuration as a disturbance propagating outward in the transverse directions from $\vec{x}_\perp=0$ at a speed of $n/\sqrt{4(N-1)+n^2}$. Since $N\geq 1$, this is less than the speed of light but approaches the speed of light for fields for which $N\ll n^2$.

Since the source for the closed string fields produced by $|\mathcal{B}_2\rangle$ is localized at $\vec{x}_\perp=0$, $|\Psi_c^{(2)}\rangle$ should satisfy source free closed string field equations away from the origin. It is easy to see that in the position space representation (3.24) is annihilated by $(L_0+\bar{L}_0)$ away from $\vec{x}_\perp=0$. Eq. (3.26) then establishes that $|\Psi_c^{(2)}\rangle$ satisfy the full set of free field equations of motion: $(Q_B+\bar{Q}_B)|\Psi_c^{(2)}\rangle=0$ away from $\vec{x}_\perp=0$.

Using (2.13) we see that $|\widehat{\mathcal{B}}\rangle_{c=1}$ and hence $|\mathcal{B}_2\rangle$ vanishes for $\lambda=\frac{1}{2}$. As a result the operators $\mathcal{O}_N^{(n)}$ defined through (2.27) vanish, and hence $|\Psi_c^{(2)}\rangle$ given in (3.24) also vanishes. Thus in the $\lambda\to\frac{1}{2}$ limit the $|\Psi_c^{(1)}\rangle$ given in (3.18) is the only contribution to the closed string background. This of course is manifestly finite in the $x^0\to\infty$ limit.

For D0-branes, and more generally for D-p-branes with all tangential directions compactified on a torus, $|\Psi_c^{(1)}\rangle$ represents a collection of massive, non-relativistic closed strings [4, 5]. The total energy density stored in $|\Psi_c^{(1)}\rangle$ turns out to be infinite. Naively this would suggest that the backreaction due to closed string emission effects invalidate the classical open string analysis of the system. However an alternative interpretation suggested in [7, 51, 8, 9, 52] is that $|\Psi_c^{(1)}\rangle$ gives the dual closed string representation of the tachyon matter predicted by the tree level open string analysis [2, 3]. The evidence for this comes from the fact that the closed string field configuration described by $|\Psi_c^{(1)}\rangle$ turns out to have properties similar to the tachyon matter predicted by the classical open string analysis. We believe that a similar interpretation must exist also for the $|\Psi_c^{(2)}\rangle$ component

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of the closed string background for a generic λ, but the lack of a complete understanding of the tree level open string results prevents us from arriving at this understanding at present. In the next few sections we shall address the same problem in two dimensional string theory where a complete understanding of the tree level open string results are available through the matrix model description of the system.

Before concluding this section we must mention that there is a different approach to constructing the boundary state of the rolling tachyon solution that gets rid of the exponentially growing terms produced by $|\mathcal{B}_2\rangle$ [53]. In this approach we begin with an appropriate boundary state in the Liouville theory with central charge > 1 where the discrete higher level primares are absent [54], analytically continue the result to the Minkowski space, and then take the $c \to 1$ limit. The final boundary state we arrive at this way has the form:

$$|\mathcal{B}\rangle_{c=1} = \int dE F(E)|E\rangle\rangle + \cdots \,, \tag{3.28}$$

where $F(E) \propto e^{-iE\ln(\sin(\pi\lambda))}/sinh(\pi E)$ is the Fourier transform of $\widetilde{f}(x^0)$, $|E\rangle\rangle$ is the Ishibashi state built on the primary $|E\rangle = \exp(-iEX^0(0))|0\rangle$, and $\cdots$ denotes the contribution from the Ishibashi states on higher level primaries at $E = 0$. Clearly this form of the boundary state does not have any exponentially growing contribution. While this could be the correct prescription, in this paper we have chosen to proceed with the prescription of computing the boundary state as well as the closed string fields produced by the boundary state in the euclidean $c = 1$ theory, and then analytically continuing it to the Minkowski theory along the branch passing through the origin of the (complex) time coordinate x^0.

4. Boundary state for the rolling tachyon in two dimensional string theory

So far our discussion has taken place in the context of critical bosonic string theory. In this section we shall study the boundary state for the rolling tachyon system in two dimensional string theory. We begin by reviewing the bulk conformal field theory associated with the two dimensional string theory. The world-sheet action of this CFT is given by the sum of three separate components:

$$s = s_L + s_{X^0} + s_{\rm ghost} \,, \tag{4.1}$$

where s_L denotes the Liouville field theory with central charge 25, s_{X^0} denotes the conformal field theory of a single scalar field X^0 describing the time coordinate and $s_{\rm ghost}$ denotes the usual ghost action involving the fields b, c, $\bar b$ and $\bar c$. Of these s_{X^0} and $s_{\rm ghost}$ are familiar objects. The Liouville action s_L on a flat world-sheet is given by:

$$s_L = \int d^2z \left(\partial_z\varphi\partial_{\bar z}\varphi + \mu e^{2\varphi}\right) \tag{4.2}$$

where φ is a world-sheet scalar field and μ is a constant parametrizing the theory. We shall set $\mu = 1$ by shifting φ by $\frac{1}{2}\ln\mu$. The scalar field φ carries a background charge $Q = 2$ (which is not visible in the flat world-sheet action (4.2) but controls the coupling of φ to the scalar curvature on a curved world-sheet), so that the theory has a central charge

$$c = 1 + 6Q^2 = 25 \,. \tag{4.3}$$

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For our analysis we shall not use the explicit world-sheet action (4.2), but only use the abstract properties of the Liouville field theory described in [55]–[59]. In particular the property of the bulk conformal field theory that we shall be using is that it has a one parameter (P) family of primary vertex operators, labelled as V_{Q+iP}, of conformal weight:

$$\left(\frac{1}{4}(Q^2+P^2), \frac{1}{4}(Q^2+P^2)\right) = \left(1+\frac{1}{4}P^2, 1+\frac{1}{4}P^2\right). \tag{4.4}$$

Thus a generic normalizable state of the bulk Liouville field theory is given by a linear combination of the secondary states built over the primary $V_{Q+iP}(0)|0\rangle$.

The closed string field $|\Psi_c\rangle$ in this two dimensional string theory is a ghost number 2 state satisfying (3.1) in the combined state space of the ghost, Liouville and X^0 field theory. If we expand $|\Psi_c\rangle$ as

$$|\Psi_c\rangle = \int \frac{dP}{2\pi} \int \frac{dE}{2\pi}\, \phi(P,E)\, c_1\bar{c}_1 e^{-iEX^0(0)} V_{Q+iP}(0)|0\rangle + \cdots, \tag{4.5}$$

then $\phi(P,E)$ is the Fourier transform of a scalar field ϕ known as the closed string 'tachyon' field.[5] Despite its name, it actually describes a massless particle in this $(1+1)$ dimensional string theory, since the condition that the state $c_1\bar{c}_1 e^{-iEX^0(0)} V_{Q+iP}(0)|0\rangle$ is on-shell is $E^2 - P^2 = 0$. Thus physically, $c\,\bar{c}\, V_{Q+iP} e^{iEX^0}$ may be regarded as the vertex operator of a scalar field ϕ of momentum P (along the Liouville direction φ) and energy E in this two dimensional string theory. This is the only physical closed string field in this theory.

We shall normalize $|\Psi_c\rangle$ so that its kinetic term is given by:

$$-\,\langle\Psi_c|c_0^-(Q_B+\bar{Q}_B)|\Psi_c\rangle\,. \tag{4.6}$$

Substituting (4.5) into (4.6) we see that the kinetic term for ϕ is given by:

$$-\frac{1}{2}\int \frac{dP}{2\pi}\frac{dE}{2\pi}\, \phi(-P,-E)(P^2-E^2)\phi(P,E)\,. \tag{4.7}$$

Thus ϕ has the standard normalization of a scalar field.

The Liouville field theory also has an unstable D0-brane obtained by putting an appropriate boundary condition on the field φ, and the usual Neumann boundary condition on the X^0 and the ghost fields. Since φ is an interacting field, it is more appropriate to describe the corresponding boundary CFT associated with the Liouville field by specifying its abstract properties. The relevant properties are as follows:

1. The open string spectrum in this boundary CFT is described by a single Virasoro module built over the $\mathrm{SL}(2,\mathbb{R})$ invariant vacuum state.

2. The one point function on the disk of the closed string vertex operator V_{Q+iP} corresponding to this boundary CFT is given by [58, 19]:

$$\langle V_{Q+iP}\rangle_D = \frac{2\,C}{\sqrt{\pi}}\, i\, \sinh(\pi P)\, \frac{\Gamma(iP)}{\Gamma(-iP)}\,, \tag{4.8}$$

where C is a normalization constant to be given in eq. (4.11).

[5] Throughout this and the next two sections we shall use the same symbol e.g. ϕ, to denote a field and its Fourier transform with respect to x^0 and/or φ coordinates.

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Since V_{Q+iP} for any real P gives the complete set of primary states in the theory, we get the boundary state associated with the D0-brane to be:[6]

$$
\begin{aligned}
|\mathcal{B}\rangle &= \frac{1}{2}\exp\left(\sum_{n=1}^{\infty}\frac{1}{n}\alpha^0_{-n}\bar{\alpha}^0_{-n}\right)|0\rangle \otimes \int\frac{dP}{2\pi}\,\langle V_{Q-iP}\rangle_D|P\rangle\rangle \otimes \\
&\quad \otimes \exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n}+b_{-n}\bar{c}_{-n})\right)(c_0+\bar{c}_0)c_1\bar{c}_1|0\rangle\,, \qquad (4.9)
\end{aligned}
$$

where $|P\rangle\rangle$ denotes the Ishibashi state in the Liouville theory, built on the primary $V_{Q+iP}(0)|0\rangle$. The normalization constant C is determined by requiring that if we eliminate $|\Psi_c\rangle$ from the combined action

$$
-\langle\Psi_c|c_0^-(Q_B+\bar{Q}_B)|\Psi_c\rangle + \langle\Psi_c|c_0^-|\mathcal{B}\rangle\,, \qquad (4.10)
$$

using its equation of motion, then the resulting value of the action reproduces the one loop open string partition function $Z_{\rm close}$ on the D0-brane. This gives [19]

$$
C = 1\,. \qquad (4.11)
$$

Given this particular boundary CFT associated with the Liouville field, we can now combine this with the rolling tachyon boundary CFT associated with the X^0 field to construct a rolling tachyon solution on the D0-brane in two dimensional string theory. As in the critical string theory, we divide the boundary state into two parts, $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_2\rangle$, with

$$
\begin{aligned}
|\mathcal{B}_1\rangle &= \frac{1}{2}\exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\alpha^0_{-n}\bar{\alpha}^0_{-n}\right)\tilde{f}(X^0(0))|0\rangle \otimes \int\frac{dP}{2\pi}\,\langle V_{Q-iP}\rangle_D\,|P\rangle\rangle \otimes \\
&\quad \otimes \exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n}+b_{-n}\bar{c}_{-n})\right)(c_0+\bar{c}_0)c_1\bar{c}_1|0\rangle \\
&\equiv \frac{1}{2}\int\frac{dP}{2\pi}\,\langle V_{Q-iP}\rangle_D\sum_N \widehat{A}_N\,\tilde{f}(X^0(0))V_{Q+iP}(0)(c_0+\bar{c}_0)c_1\bar{c}_1|0\rangle\,, \qquad (4.12)
\end{aligned}
$$

and

$$
\begin{aligned}
|\mathcal{B}_2\rangle &= \frac{1}{2}|\widetilde{\mathcal{B}}\rangle_{c=1} \otimes \int\frac{dP}{2\pi}\,\langle V_{Q-iP}\rangle_D\,|P\rangle\rangle \otimes \\
&\quad \otimes \exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n}+b_{-n}\bar{c}_{-n})\right)(c_0+\bar{c}_0)c_1\bar{c}_1|0\rangle \\
&\equiv \frac{1}{2}\sum_{n\in Z}\sum_{N=1}^{\infty}\int\frac{dP}{2\pi}\,\langle V_{Q-iP}\rangle_D\,\widehat{\mathcal{O}}^{(n)}_N\,(c_0+\bar{c}_0)\,c_1\,\bar{c}_1\,e^{nX^0(0)}\,V_{Q+iP}(0)\,|0\rangle\,. \qquad (4.13)
\end{aligned}
$$

Here $|\widehat{\mathcal{B}}\rangle_{c=1}$ is the inverse Wick rotated version of $|\widetilde{B}\rangle_{c=1}$ as defined in eq. (2.7), and $\widehat{A}_N$ and $\widehat{\mathcal{O}}^{(n)}_N$ are operators of level (N,N), consisting of non-zero mode ghost and X^0 oscillators, and the Virasoro generators of the Liouville theory.

[6] The normalization factor of 1/2 is a reflection of the fact that if we take $|\Psi_c\rangle = c_1\bar{c}_1\,V_{Q+iP}(0)|0\rangle$ and calculate $\langle\Psi_c|c_0^-|\mathcal{B}\rangle$ we get a factor of 2 in the ghost correlator $\langle 0|c_{-1}\bar{c}_{-1}c_0^-c_0^+c_1\bar{c}_1|0\rangle$.

As in the case of critical string theory, it is easy to show that $|\mathcal{B}_1\rangle$ and $\mathcal{B}_2\rangle$ are separately BRST invariant. (4.12) and (4.13) shows that the sources for the various closed string fields in the momentum space are proportional to $\langle V_{Q-iP}\rangle_D$. It is instructive to see what they correspond to in the position space labelled by the Liouville coordinate φ. We concentrate on the negative φ region since for large negative φ the effect of the $e^{2\varphi}$ term in (4.2) is small and the Liouville coordinate behaves like a free scalar field on the world-sheet. Thus V_{Q+iP} takes the form $e^{(Q+iP)\varphi} = e^{2\varphi+iP\varphi}$, and

$$|P\rangle\rangle \sim \widehat{\mathcal{O}}_L\, e^{2\varphi(0)+iP\varphi(0)}|0\rangle\,, \tag{4.14}$$

where $\widehat{\mathcal{O}}_L$ is an appropriate operator in the Liouville field theory. In this region the source term becomes proportional to

$$\int \frac{dP}{2\pi}\, e^{2\varphi+iP\,\varphi}\langle V_{Q-iP}\rangle_D \propto \int \frac{dP}{2\pi}\, e^{2\varphi+iP\,\varphi}\;\sinh(\pi P)\,\frac{\Gamma(-iP)}{\Gamma(iP)}\,. \tag{4.15}$$

As it stands the integral is not well defined since $\sinh(\pi P)$ blows up for large $|P|$. For negative φ, we shall define this integral by closing the contour in the lower half plane, and picking up the contribution from all the poles enclosed by the contour. Since the poles of $\Gamma(-iP)$ at $P=-in$ are cancelled by the zeroes of $\sinh(\pi P)$ we see that the integrand has no pole in the lower half plane and hence the integral vanishes. Thus the boundary state $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_2\rangle$ given in (4.12) and (4.13) do not produce any source term for negative φ. This in turn leads to the identification of this system as a D0-brane that is localized in the Liouville direction [19].

The same argument as in the case of critical string theory indicates that $|\mathcal{B}_2\rangle$ encodes information about conserved charges. To see explicitly what these conserved charges correspond to, we first express $|\mathcal{B}_2\rangle$ in a manner similar to that in (2.29)

$$|\mathcal{B}_2\rangle = \frac{1}{2}\sum_{j=1}^{\infty}\;\sum_{m=-(j-1)}^{j-1}\int\frac{dP}{2\pi}\,\langle V_{Q-iP}\rangle_D\, f_{j,m}(\lambda)\,\widehat{\mathcal{R}}^{(2d)}_{j,m}\,(c_0+\bar{c}_0)c_1\bar{c}_1 e^{2mX^0(0)}\,|k^0=0,P\rangle\,, \tag{4.16}$$

where

$$\widehat{\mathcal{R}}^{(2d)}_{j,m} = \widehat{\mathcal{N}}_{j,m}\,\widehat{\mathcal{O}}_L\,\exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n}+b_{-n}\bar{c}_{-n})\right), \tag{4.17}$$

and $f_{j,m}(\lambda)$ and $\widehat{\mathcal{N}}_{j,m}$ are as defined in eq. (2.12) and (2.31) respectively. If we now generalize the source term so that it has the same operator structure but arbitrary time dependence:

$$|\mathcal{B}_2\rangle' = \frac{1}{2}\sum_{j=1}^{\infty}\;\sum_{m=-(j-1)}^{j-1}\int\frac{dP}{2\pi}\,\langle V_{Q-iP}\rangle_D\,\widehat{\mathcal{R}}^{(2d)}_{j,m}\,(c_0+\bar{c}_0)c_1\bar{c}_1\, g_{j,m}(X^0(0))\,|k^0=0,P\rangle\,, \tag{4.18}$$

then requiring $(Q_B+\bar{Q}_B)|\mathcal{B}_2\rangle'=0$ gives:

$$\partial_0\left(e^{-2mx^0}g_{j,m}(x^0)\right)=0\,, \tag{4.19}$$

as in the case of critical string theory. Thus $e^{-2mx^0}g_{j,m}(x^0)$ can be thought of as a conserved charge which takes value $f_{j,m}(\lambda)$ for $|\mathcal{B}_2\rangle$ given in (4.16).

These charges clearly bear a close relation to the global symmetries of this two dimensional string theory discussed in [36, 37] but we shall not explore this relation here. In section 6 we shall directly relate these charges to the conserved charges in the matrix model description of this theory, which, in turn, are known to be related to the global symmetries discussed in [36, 37].

5. Closed string background produced by the rolling tachyon in two dimensional string theory

We now calculate the closed string field produced by this time dependent boundary state. The contribution from the $|\mathcal{B}_1\rangle$ part of the boundary state can be easily computed as in the case of critical string theory, and in the $x^0 \to \infty$ limit takes the form:

$$|\Psi_c^{(1)}\rangle = \int \frac{dP}{2\pi} \langle V_{Q-iP}\rangle_D \sum_N \widehat{A}_N \, h_{\vec{k}_\perp}^{(N+1)}(X^0(0)) V_{Q+iP}(0) c_1 \bar{c}_1 |0\rangle \,, \tag{5.1}$$

where $h_{\vec{k}_\perp}^{(N)}(x^0)$ has been defined in (3.15). The $(N+1)$ in the superscript of h in (5.1) can be traced to the fact that in this theory a level (N, N) state has mass$^2 = 4N$ whereas in the critical string theory a level (N, N) state had mass$^2 = 4(N-1)$.

Arguments similar to those given for critical string theory shows that in the $x^0 \to \infty$ limit this state is on-shell, i.e. it is annihilated by the BRST charge $(Q_B + \bar{Q}_B)$. Since the only physical states in the theory come from the closed string tachyon state, it must be possible to remove all the other components of $|\Psi_c^{(1)}\rangle$ by an on shell gauge transformation of the form $\delta|\Psi_c\rangle = (Q_B + \bar{Q}_B)|\Lambda\rangle$ by suitably choosing $|\Lambda\rangle$. Furthermore, since the action of Q_B and $\bar{Q}_B$ does not mix states of different levels, it must be possible to remove all the $N > 0$ components of $|\Psi_c^{(1)}\rangle$ without modifying the $N = 0$ component. Using the expression for $h_{\vec{k}_\perp}^{(1)}$ from (3.16), (3.17), and $\langle V_{Q-iP}\rangle_D$ from (4.8) we get the following expression for the closed string tachyon field ϕ in the $x^0 \to \infty$ limit [19]

$$\begin{aligned} \phi(P, x^0 \to \infty) \; = & -\frac{\pi}{\sinh(\pi\omega_P)} \frac{1}{2\omega_P} \frac{2}{\sqrt{\pi}} i \sinh(\pi P) \frac{\Gamma(-iP)}{\Gamma(iP)} \times \\ & \times \left[e^{-i\omega_P(x^0 + \ln\sin(\pi\lambda))} + e^{i\omega_P(x^0+\ln\sin(\pi\lambda))} \right] , \end{aligned} \tag{5.2}$$

where $\omega_P = |P|$ since we are dealing with a single massless scalar particle. This can be simplified as

$$\phi(P, x^0 \to \infty) = -i \frac{\sqrt{\pi}}{P} \frac{\Gamma(-iP)}{\Gamma(iP)} \left[e^{-iP(x^0+\ln\sin(\pi\lambda))} + e^{iP(x^0+\ln\sin(\pi\lambda))} \right] . \tag{5.3}$$

This finishes our discussion of closed string radiation induced by the $|\mathcal{B}_1\rangle$ component of the boundary state.

We shall now discuss the closed string background produced by $|\mathcal{B}_2\rangle$. This can be analyzed in the same way as in the case of critical string theory. We begin with the expression (4.13) of $|\mathcal{B}_2\rangle$. Since $\widehat{\mathcal{O}}_N^{(n)} \, (c_0 + \bar{c}_0) \, c_1 \, \bar{c}_1 \, e^{nX^0(0)} \, V_{Q+iP}(0) \, |0\rangle$ in this expression is an eigenstate of $2(L_0 + \bar{L}_0)$ with eigenvalue $(4N + n^2 + P^2)$, we can choose the closed string field produced by $|\mathcal{B}_2\rangle$ to be:

$$|\Psi_c^{(2)}\rangle = \sum_{n\in Z}\sum_{N=2}^{\infty} \int \frac{dP}{2\pi} \langle V_{Q-iP}\rangle_D \left(4N + n^2 + P^2\right)^{-1} \widehat{\mathcal{O}}_N^{(n)} \, c_1 \, \bar{c}_1 \, e^{nX^0(0)} \, V_{Q+iP}(0) \, |0\rangle \, . \tag{5.4}$$

This corresponds to closed string field configurations which grow as e^{nx^0} for large x^0.

A special class of operators among the $\widehat{O}_N^{(n)}$'s are those which involve only excitations involving the α^0, $\bar{\alpha}^0$ oscillators and correspond to higher level *primaries* of the $c = 1$ conformal field theory. As described before, these primaries are characterized by SU(2) quantum numbers (j, m) with $j \geq 1$, $-j < m < j$, and has dimension (j^2, j^2). The quantum number m can be identified as $n/2$ in (5.4). From (2.9), (2.31), (4.16), and (4.17) we see that the contribution to $|\mathcal{B}_2\rangle$ from these primary states has the form:

$$\frac{1}{2}\sum_{j\geq 1}\sum_{m=-j+1}^{j-1} \int \frac{dP}{2\pi} \langle V_{Q-iP}\rangle_D \, f_{j,m}(\lambda) \, \widehat{\mathcal{P}}_{j,m} \, (c_0 + \bar{c}_0) \, c_1 \, \bar{c}_1 \, e^{2mX^0(0)} \, V_{Q+iP}(0) \, |0\rangle \, . \tag{5.5}$$

The level of the operators $\widehat{\mathcal{P}}_{j,m}$ is

$$N = (j^2 - m^2) \, . \tag{5.6}$$

Thus the $|\Psi_c^{(2)}\rangle$ produced by this part of $|\mathcal{B}_2\rangle$ takes the form:

$$|\check{\Psi}_c^{(2)}\rangle = \sum_{j,m} f_{j,m}(\lambda) \int \frac{dP}{2\pi} \langle V_{Q-iP}\rangle_D \, (4j^2 + P^2)^{-1} \, \widehat{\mathcal{P}}_{j,m} \, c_1 \, \bar{c}_1 \, e^{2mX^0(0)} \, V_{Q+iP}(0) \, |0\rangle \, . \tag{5.7}$$

As in the case of critical string theory, it is instructive to study the behaviour of $|\Psi_c^{(2)}\rangle$ in the position space characterized by the Liouville coordinate φ instead of the momentum space expression given in (5.4). We concentrate on the negative φ region since for large negative φ the effect of the $e^{2\varphi}$ term in (4.2) is small and the Liouville coordinate behaves like a free scalar field on the world-sheet. Let us first focus on the $|\check{\Psi}_c^{(2)}\rangle$ part of $|\Psi_c^{(2)}\rangle$ as given in (5.7). In the position space the string field component associated with the state $\widehat{\mathcal{P}}_{j,m} \, c_1 \, \bar{c}_1 |E, P\rangle$ is given by

$$\psi_{j,m}(\varphi, x^0) = f_{j,m}(\lambda) \, e^{2mx^0} \int \frac{dP}{2\pi} \, e^{2\varphi + iP\,\varphi} \, (4j^2 + P^2)^{-1} \, \langle V_{Q-iP}\rangle_D \, . \tag{5.8}$$

Using the expression for $\langle V_{Q+iP}\rangle_D$ given in (4.8) we get

$$\psi_{j,m}(\varphi, x^0) = -\frac{2}{\sqrt{\pi}} \, i \, f_{j,m}(\lambda) \, e^{2mx^0} \int \frac{dP}{2\pi} \, e^{2\varphi + iP\,\varphi} \, (4j^2 + P^2)^{-1} \, \sinh(\pi P) \, \frac{\Gamma(-iP)}{\Gamma(iP)} \, . \tag{5.9}$$

This integral is not well defined since $\sinh(\pi P)$ blows up for large $|P|$. As in the analysis of (4.15), for negative φ we shall define this integral by closing the contour in the lower half plane, and picking up the contribution from all the poles. Since the poles of $\Gamma(-iP)$

at $P = -in$ are cancelled by the zeroes of $\sinh(\pi P)$, the only pole that the integral has in the lower half plane is at $P = -2ij$. Evaluating the residue at this pole, we get

$$\psi_{j,m}(\varphi, x^0) = \sqrt{\pi}\, f_{j,m}(\lambda)\, e^{2mx^0+2(1+j)\varphi} \frac{1}{((2j)!)^2}\,. \tag{5.10}$$

In the language of string field theory, this corresponds to

$$|\check{\Psi}_c^{(2)}\rangle = \sum_{j,m} \frac{\sqrt{\pi}}{((2j)!)^2}\, f_{j,m}(\lambda)\, \widehat{\mathcal{P}}_{j,m}\, e^{2mX^0(0)}|0\rangle_{X^0} \otimes V_{2(1+j)}(0)|0\rangle_L \otimes c_1\, \bar{c}_1|0\rangle_{\rm ghost}\,. \tag{5.11}$$

The states appearing in (5.11) are precisely the discrete states of two dimensional string theory [35, 36] (after inverse Wick rotation $X \to iX^0$.)

Let us now turn to analyzing the contribution from the rest of the terms in $|\mathcal{B}_2\rangle$. From the general formula (5.4) we see that this will correspond to linear combination of states created from $\widehat{\mathcal{P}}_{j,m}\, c_1\, \bar{c}_1 e^{2mX^0(0)} V_{Q+iP}(0)|0\rangle$ by the action of ghost oscillators and the Virasoro generators of the X^0 and the Liouville field theory. Furthermore by following the same argument as in the case of critical string theory we can show that this field configuration must be on-shell, i.e. annihilated by $(Q_B + \bar{Q}_B)$. The BRST cohomology analysis of [35] then tells us that these states must be BRST trivial, since the only non-trivial elements of the BRST cohomology in the ghost number two sector are obtained by taking products of primary states in the matter and the Liouville sector with the ground state $c_1\bar{c}_1|0\rangle$ of the ghost sector. Thus the part of $|\Psi_c^{(2)}\rangle$ other than the one given in (5.11) can be removed by a gauge transformation $\delta|\Psi_c\rangle = (Q_B + \bar{Q}_B)|\Lambda\rangle$. Furthermore, since Q_B and $\bar{Q}_B$ do not mix levels, the gauge transformations which remove higher level states built on matter and Liouville primaries cannot affect the part of the string field given in (5.11). Thus we conclude that up to a gauge transformation, the effect of $|\mathcal{B}_2\rangle$ is to produce the on-shell string field configuration given in (5.11).[7]

6. Matrix model description of two dimensional string theory

The two dimensional string theory described above also has an alternative description to all orders in perturbation theory as a matrix model [15, 16, 17]. This matrix description, in turn, can be shown to be equivalent to a theory of infinite number of non-interacting fermions, each moving in an inverted harmonic oscillator potential with hamiltonian

$$h(p,q) = \frac{1}{2}(p^2 - q^2) + \frac{1}{g_s}\,, \tag{6.1}$$

[7] Since earlier we had argued that all contributions to $|\Psi_c^{(1)}\rangle$ other than the one due to the closed string tachyon field can be removed by a gauge transformation, one might wonder why we cannot also remove $|\widetilde{\Psi}_c^{(2)}\rangle$ given in (5.11) by a gauge transformation. The reason for this is that the states with exponential time dependence were not included in the BRST cohomology analysis which led to the conclusion that all contribution to $|\Psi_c^{(1)}\rangle$ other than the ones coming from the tachyon are BRST trivial. This assumption was justified for $|\Psi_c^{(1)}\rangle$ which did not have any exponential time dependence, but is clearly not justified for $|\widetilde{\Psi}_c^{(2)}\rangle$.

where (q,p) denote a canonically conjugate pair of variables. The coordinate variable q is related to the eigenvalue of an infinite dimensional matrix, but this information will not be necessary for our discussion. Clearly $h(p,q)$ has a continuous energy spectrum spanning the range $(-\infty,\infty)$. The vacuum of the theory corresponds to all states with negative h eigenvalue being filled and all states with positive h eigenvalue being empty. Thus the fermi surface is the surface of zero energy. Since we shall not go beyond perturbation theory, we shall ignore the effect of tunneling from one side of the barrier to the other side and work on only one side of the barrier. For definiteness we shall choose this to be the negative q side. In the semi-classical limit, in which we represent a quantum state by an area element of size $\hbar$ in the phase space spanned by p and q, we can restrict ourselves to the negative q region, and represent the vacuum by having the region $(p^2-q^2)\le -\frac{2}{g_s}$ filled, and rest of the region empty [60, 61]. Thus in this picture the fermi surface in the phase space corresponds to the curve:

$$\frac{1}{2}(p^2-q^2)+\frac{1}{g_s}=0\,. \tag{6.2}$$

If $\Psi(q,t)$ denotes the second quantized fermion field describing the above non-relativistic system, then the massless 'tachyon' field in the closed string sector is identified with the scalar field obtained by the bosonization of the fermion field Ψ [30, 31, 32]. The precise correspondence goes as follows. The classical equation of motion satisfied by the field $\Psi(q,x^0)$ has the form:

$$i\frac{\partial\Psi}{\partial x^0}+\frac{1}{2}\frac{\partial^2\Psi}{\partial q^2}+\frac{1}{2}q^2\Psi-\frac{1}{g_s}\Psi=0\,. \tag{6.3}$$

We now define the 'time of flight' variable τ that is related to q via the relation:

$$q=-\sqrt{\frac{2}{g_s}}\cosh\tau\,,\qquad \tau<0\,. \tag{6.4}$$

$|\tau|$ measures the time taken by a zero energy classical particle moving under the hamiltonian (6.1) to travel from $-\sqrt{\frac{2}{g_s}}$ to q. We also define

$$v(q)=-\sqrt{q^2-\frac{2}{g_s}}=\sqrt{\frac{2}{g_s}}\sinh\tau\,. \tag{6.5}$$

$|v(q)|$ gives the classical velocity of a zero energy particle when it is at position q. Using these variables, it is easy to see that for large negative τ the solution to eq. (6.3) takes the form:

$$\Psi(q,x^0)=\frac{1}{\sqrt{-2v(q)}}\left[e^{-i\int^q v(q')dq'+i\pi/4}\,\Psi_R(\tau,x^0)+e^{i\int^q v(q')dq'-i\pi/4}\,\Psi_L(\tau,x^0)\right], \tag{6.6}$$

where Ψ_L and Ψ_R satisfy the field equations:

$$(\partial_0-\partial_\tau)\,\Psi_L(\tau,x^0)=0\,,\qquad (\partial_0+\partial_\tau)\,\Psi_R(\tau,x^0)=0\,. \tag{6.7}$$

Thus at large negative τ we can regard the system as a theory of a pair of chiral fermions, one left-moving and the other right-moving. Of course there is an effective boundary condition at $\tau = 0$ which relates the two fermion fields, since a particle coming in from $\tau = -\infty$ will be reflected from $\tau = 0$ and will go back to $\tau = -\infty$. Since τ ranges from 0 to $-\infty$, we can interprete Ψ_R as the incoming wave and Ψ_L as the outgoing wave.

Since Ψ_L and Ψ_R represent a pair of relativistic fermions, we can bosonize them into a pair of chiral bosons χ_L and χ_R. This pair of chiral bosons may in turn be combined into a full scalar field $\chi(\tau, x^0)$ which satisfy the free field equation of motion for large τ but has a complicated boundary condition at $\tau = 0$. If χ is defined with the standard normalization, then for large τ a single right moving fermion is represented by the configuration [62][8]

$$\chi = \sqrt{\pi}\, H(x^0 - \tau)\,, \tag{6.8}$$

and a single left-moving fermion is represented by the configuration

$$\chi = \sqrt{\pi}\, H(x^0 + \tau)\,, \tag{6.9}$$

where H denotes the step function:

$$H(u) = \begin{cases} 1 & \text{for} \quad u > 0 \\ 0 & \text{for} \quad u < 0\,. \end{cases} \tag{6.10}$$

The field $\chi(\tau, x^0)$ is related to the tachyon field $\phi(\varphi, x^0)$ in the continuum description of string theory by a non-local field redefinition. This relation is easy to write down in the momentum space. If $\chi(p, x^0)$ denotes the Fourier transform of χ with respect to the variables τ, with p denoting the momentum variable conjugate to τ, and $\phi(P, x^0)$ denotes the Fourier transform of ϕ with respect to the Liouville coordinate φ, with P denoting the momentum variable conjugate to φ, then we have [63, 64]:

$$\chi(P, x^0) = \frac{\Gamma(iP)}{\Gamma(-iP)}\, \phi(P, x^0)\,. \tag{6.11}$$

As a result, the background χ corresponding to the ϕ field configuration given in (5.3) is given by:

$$\chi(P, x^0) = -i\, \frac{\sqrt{\pi}}{P} \left[e^{-iP(x^0 + \ln\sin(\pi\lambda))} + e^{iP(x^0 + \ln\sin(\pi\lambda))} \right]. \tag{6.12}$$

In τ space this corresponds to the background:

$$\chi(\tau, x^0) = -\sqrt{\pi}[H(x^0 + \ln\sin(\pi\lambda) - \tau) - H(x^0 + \ln\sin(\pi\lambda) + \tau)] + \text{constant}\,. \tag{6.13}$$

Eq. (6.13) is valid only in the $x^0 \to \infty$ limit. Since $\tau < 0$, in this limit the first term goes to a constant which can be removed by a redefinition of χ by a constant shift, and we get

$$\chi(\tau, x^0) = \sqrt{\pi}\, H(x^0 + \tau + \ln\sin(\pi\lambda))\,. \tag{6.14}$$

[8]Such a configuration has infinite energy at the classical level in the scalar field theory. In the fermionic description this infinite energy is the result of infinite quantum uncertainty in momentum for a sharply localized particle in the position space. Thus the classical limit of the fermionic theory does not have this infinite energy. This is the origin of the apparent discrepancy between the classical open string calculation of the D0-brane energy which gives a finite answer and the classical closed string calculation which gives infinite answer [19].

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According to (6.9) this precisely represents a single left-moving (outgoing) fermion. This shows that the non-BPS D0-brane of the two dimensional string theory can be identified as a state of the matrix theory where a single fermion is excited from the fermi level to some energy > 0 [18, 20, 19].

Since the fermions are non-interacting, the states with a single excited fermion do not mix with any other states in the theory (say with states where two or more fermions are excited above the fermi level or states where a fermion is excited from below the fermi level to the fermi level). As a result, the quantum states of a D0-brane are in one to one correspondence with the quantum states of the single particle hamiltonian $h(p, q)$ given in (6.1) with one additional constraint, — the spectrum is cut off sharply for energy below zero due to Pauli exclusion principle. Thus in the matrix model description, the quantum 'open string field theory' for a single D0-brane is described by the inverted harmonic oscillator hamiltonian (6.1) with all the negative energy states removed by hand [9]. The classical limit of this quantum hamiltonian is described by the classical hamiltonian (6.1), with a sharp cut-off on the phase space variables:

$$\frac{1}{2}(p^2 - q^2) + \frac{1}{g_s} \geq 0\,. \tag{6.15}$$

This is the matrix model description of classical 'open string field theory' describing the dynamics of a D0-brane. In this description the D0-brane with the tachyon field sitting at the maximum of the potential corresponds to the configuration $p = 0$, $q = 0$. The mass of the D0-brane is then given by $h(0, 0) = 1/g_s$.

Clearly the quantum system described above provides us with a complete description of the dynamics of a single D0-brane. In particular there is no need to couple this system explicitly to closed strings, although at late time closed strings provide an alternative description of the D0-brane as a kink solution (given in (6.14)). This is in accordance with the general open-closed string duality conjecture. In the classical limit the rolling tachyon solution in open string theory, characterized by the parameter λ, corresponds to the phase space trajectory

$$\frac{1}{2}(p^2 - q^2) + \frac{1}{g_s} = \frac{1}{g_s}\cos^2(\pi\lambda)\,, \tag{6.16}$$

as can be seen by comparing the energies of the rolling tachyon system [3] and the system described by the hamiltonian (6.1). In particular the $\lambda \to \frac{1}{2}$ limit corresponds to a trajectory at the fermi level.

From this discussion it is clear that it is a wrong notion to think in terms of *backreaction of closed string fields on the open string dynamics*. Instead we should regard the closed string background produced by the D-brane as a way of characterizing the open string background (although the open string theory itself is sufficient for this purpose). For example, in the present context, we can think of the closed string tachyon field $\chi(\tau, x^0)$ at late time as the expectation value of the operator[9]

$$\hat{\chi}(\tau, x^0) \equiv \sqrt{\pi}\, H\left(-\hat{q}(x^0) - \sqrt{\frac{2}{g_s}}\cosh\tau\right) \tag{6.17}$$

in the quantum open string theory on a single D0-brane, as described by (6.1), (6.15). In (6.17) $\hat{q}$ denotes the position operator in the quantum open string theory. When we calculate the expectation value of $\hat{\chi}$ in the quantum state whose classical limit is described by the trajectory (6.16), we can replace $\widehat{q}$ by its classical value $q = -\sqrt{\frac{2}{g_s}}\sin(\pi\lambda)\cosh(x^0)$. This gives

$$\langle\hat{\chi}(\tau, x^0)\rangle = \sqrt{\pi}\, H\left(\sqrt{\frac{2}{g_s}}\sin(\pi\lambda)\,\cosh(x^0) - \sqrt{\frac{2}{g_s}}\cosh\tau\right) \simeq \sqrt{\pi}\, H(x^0 + \tau + \ln\sin(\pi\lambda))\,, \tag{6.18}$$

for large x^0 and negative τ. This precisely reproduces (6.14).

Given the interpretation (6.18) of the closed string field $|\Psi_c^{(1)}\rangle$ produced by the $|\mathcal{B}_1\rangle$ component of the boundary state, we can now ask if it is possible to find similar interpretation for the exponentially growing component $|\Psi_c^{(2)}\rangle$ of the string field produced by $|\mathcal{B}_2\rangle$. We can begin with the simpler task of trying to identify the conserved charges $e^{-2mx^0} g_{j,m}(x^0)$ associated with $|\mathcal{B}_2\rangle$. An infinite set of conserved charges of this type do indeed exist in the quantum theory of a single fermion described by (6.1). These are of the form [31, 38, 39, 36, 40, 41, 42, 43]:

$$e^{-(k-l)x^0}(p+q)^k(q-p)^l\,, \tag{6.19}$$

where k and l are integers. Requiring that the canonical transformations generated by these charges preserve the constraint (6.15) [36] gives us a more restricted class of charges:

$$h(p,q)\, e^{-(k-l)x^0}(p+q)^k(q-p)^l = \left(\frac{1}{2}(p^2 - q^2) + \frac{1}{g_s}\right) e^{-(k-l)x^0}(p+q)^k(q-p)^l\,. \tag{6.20}$$

Thus it is natural to identify these with linear combinations of the charges $e^{-2mx^0} g_{j,m}(x^0)$ in the continuum theory. In order to find the precise relation between these charges we can first compare the explicit x^0 dependence of the two sets of charges. This gives:

$$k - l = 2m\,. \tag{6.21}$$

Thus the conserved charge $e^{-2mx^0} g_{j,m}(x^0)$ should correspond to some specific linear combination of the charges given in (6.20) subject to the condition (6.21):[10]

$$g_{j,m}(x^0) \leftrightarrow g_s\left(\frac{1}{2}(p^2 - q^2) + \frac{1}{g_s}\right)\sum_{\substack{k\in Z\\ k\geq 0,2m}}\left(\frac{2}{g_s}\right)^{m-k} a_k^{(j,m)}\,(p+q)^k(q-p)^{k-2m}\,. \tag{6.22}$$

[9] $\partial_\tau\hat{\chi}$ is the representation of the usual density operator of free fermions in the Hilbert space of first quantized theory of a single fermion.

[10] Ref. [14] proposed an alternative route to relating the parameter λ labelling the rolling tachyon boundary state to the parameter labelling the matrix model solutions through the ground ring generators [36]. However, since the ground ring generators are operators of ghost number zero, their expectation value on the disk vanishes by ghost charge conservation, and hence they have vanishing inner product with the boundary state. Due to this reason the relationship between the analysis of [14] and that given here is not quite clear.

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Here $a_k^{(j,m)}$ are constants and the various g_s dependent normalization factors have been introduced for later convenience. In order to find the precise form of the coefficients $a_k^{(j,m)}$ we compare the λ dependence of the two sides for the classical trajectory (6.16). Since for this trajectory

$$q \pm p = -\sqrt{\frac{2}{g_s}}\,\sin(\pi\lambda)\, e^{\pm x^0}\,, \tag{6.23}$$

and $g_{j,m}(x^0) = e^{2mx^0}\, f_{j,m}(\lambda)$, we have:

$$f_{j,m}(\lambda) = (-1)^{2m}\, \left(1 - \sin^2(\pi\lambda)\right) \sum_{\substack{k\in Z \\ k\geq 0,2m}} a_k^{(j,m)}\, \sin^{2k-2m}(\pi\lambda)\,. \tag{6.24}$$

Thus by expanding $f_{j,m}(\lambda)$ given in (2.12) in powers of $\sin(\pi\lambda)$ we can determine the coefficients $a_k^{(j,m)}$. One consistency check for this procedure is that on the right hand side the expansion in powers of $\sin(\pi\lambda)$ starts at order $\sin^{2|m|}(\pi\lambda)$. It can be verified that the expansion of $f_{j,m}(\lambda)$ also starts at the same order. The other consistency check is that the right hand side of (6.24) vanishes at $\lambda = 1/2$, which is also the case for $f_{j,m}(\lambda)$.

For the purpose of illustration we quote here the non-zero values of $a_k^{(1,0)}$ and $a_k^{(3/2,1/2)}$ using (2.15):

$$a_0^{(1,0)} = -2\,, \qquad a_1^{(3/2,1/2)} = -3\,. \tag{6.25}$$

In general it follows from the definition (2.12) of $f_{j,m}(\lambda)$ and the properties of $D^j_{m,-m}(\theta)$ that as a power series expansion in $\sin(\pi\lambda)$ the maximum power of $\sin(\pi\lambda)$ that can appear in the expression for $f_{j,m}(\lambda)$ is $2j$. Thus the sum over k in (6.24) must be restricted to

$$k \leq j + m - 1\,. \tag{6.26}$$

In other words, $a_k^{(j,m)}$ vanishes for $k > j+m-1$. We shall now show using this fact that the relations (6.22) are invertible, *i.e.* we can solve them to express $h(p,q)(p+q)^k(q-p)^{k-2m}$ in terms of the $g_{j,m}$'s. For this let us restrict to the case $m \geq 0$; the $m < 0$ case may be analysed in a similar fashion. In this case the sum over k in (6.22) runs from $2m$ to $j + m - 1$. Thus for $j = m + 1$, the only term in the sum is $k = 2m$. This gives:

$$g_{m+1,m}(x^0) = g_s\, \left(\frac{g_s}{2}\right)^m\, a_{2m}^{(m+1,m)}\, h(p,q)\, (q+p)^{2m}\,. \tag{6.27}$$

This expresses $h(p,q)\,(q+p)^{2m}$ in terms of $g_{m+1,m}(x^0)$. Now taking $j = m+2$ in (6.22), we can express $g_{m+2,m}(x^0)$ as a linear combination of $h(p,q)\,(q+p)^{2m}$ and $h(p,q)\,(q+p)^{2m+1}\,(q-p)$. From this and (6.27) we get $h(p,q)\,(q+p)^{2m+1}\,(q-p)$ in terms of $g_{m+1,m}(x^0)$ and $g_{m+2,m}(x^0)$. Repeating this process we see that in general $h(p,q)\,(p+q)^{2m+l}\,(q-p)^l$ may be expressed as a linear combination of $g_{j,m}(x^0)$ for $m+1 \leq j \leq m+l+1$. This shows that the conserved charges $g_{j,m}(x^0)$ in the continuum theory contains information about the complete set of symmetry generators in the matrix model description of the D0-brane.

Since the fields produced by these sources are also proportional to $e^{2mx^0} f_{j,m}(\lambda)$, we expect that the matrix model representation of these fields also involve the same combination as (6.22). For example we can consider the operator:

$$\hat{\chi}_{j,m}(\tau, x^0) \sim g_s\, H\left(-\hat{q}(x^0) - \sqrt{\frac{2}{g_s}}\cosh\tau\right)\left(\frac{1}{2}(\hat{p}(x^0)^2 - \hat{q}(x^0)^2) + \frac{1}{g_s}\right) \times \\ \times \sum_{k\geq 0,2m}^{j+m-1}\left(\frac{2}{g_s}\right)^{m-k} a_k^{(j,m)}\,(\hat{p}(x^0) + \hat{q}(x^0))^k(\hat{q}(x^0) - \hat{p}(x^0))^{k-2m}\,. \tag{6.28}$$

In this case at late time $\langle\partial_\tau\, \hat{\chi}_{j,m}(\tau, x^0)\rangle$ for the classical trajectory (6.16) behaves as

$$f_{j,m}(\lambda)\, e^{2mx^0}\delta(x^0 + \tau + \ln\sin(\pi\lambda))\,. \tag{6.29}$$

This of course has the same λ and x^0 dependence as $\psi_{j,m}(\varphi, x^0)$ given in (5.10), but it is localized in τ space at $-(x^0+\ln\sin(\pi\lambda))$ by the δ function instead of having the exponential tail of $\psi_{j,m}(\varphi, x^0)$ in the negative φ region. This however is not necessarily a contradiction, since the fields $\langle\hat{\chi}_{j,m}(\tau, x^0)\rangle$ are likely to be related to the fields $\psi_{j,m}(\varphi, x^0)$ by a non-local transformation similar to the one given in (6.11). Such non-local transformations are known to produce exponential tails in the fields in the position space representation [64].

In the context we note that by carefully examining the bosonization rules for a fermion moving under the influence of inverted harmonic oscillator potential, ref. [29] has recently argued that in order to describe the motion of a single fermion in the language of closed string theory, we need to switch on infinite number of closed string fields besides the tachyon. We believe that the presence of the additional closed string background (5.10), (5.11) associated with the discrete states is a reflection of this effect. As a consistency check we note that at $\lambda = 1/2$ the additional background (5.11) vanish. This is expected to be true in the matrix model description as well [29].

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